The Welfare Effects of Fiscal Equalisation[1]

Jeffrey D. Petchey[2]

Abstract

This paper shows that schemes of fiscal equalisation, common in many federations, may lead to inefficient transfers of income between regions, sub-optimal provision of local public goods and unambiguously lower social welfare without achieving their equity objectives. Reforms that make the transfers depend more on efficiency-based factors, such as fiscal externalities, location specific economic rents and congestion costs, have the potential to enhance social welfare.

Keywords: inter-State transfers, fiscal externalities, fiscal equalisation, Nash equilibrium, labour mobility, strategic behaviour, public goods, efficiency, social welfare, efficiency frontier.

---

[1] This paper presents results of work in progress on the efficiency effects of fiscal equalisation and should not be reproduced or quoted without permission from the author.

[2] Address for correspondence: *Fiscal Federalism Research Programme*, School of Economics and Finance, Curtin University of Technology, GPO Box U 1987, Perth, Western Australia, Australia. Phone (international): (61) 8 9266 7408. Fax (international): (61) 8 9266 3026. Email: petcheyj@cbs.curtin.edu.au. The research contained in this paper was conducted under the auspices of an Australian Research Council Large Grant for work on Australia's system of fiscal equalisation. The author would like to thank the ARC for its support. Thanks are also due to Perry Shapiro and Sofia Levtchenkova for extremely helpful comments on earlier drafts.

1. Introduction

Many federal systems have nationally mandated 'fiscal equalisation' schemes that result in substantial transfers between sub-national governments. Mostly, these schemes are motivated by a concern to achieve equity goals within the federation, rather than efficiency. For example, the Australian, Canadian and Swiss schemes of equalisation, which are particular forms of revenue sharing, consist of a set of formulas that determine the inter-State distribution of a centrally collected pool of revenue. They seek to distribute the revenue pool in such a way that reduces regional disparities. The Australian system, arguably the most comprehensive of all, attempts to estimate both expenditure and revenue 'needs' of the States, and to direct more funds to those States with relatively higher needs. The Canadian system, on the other hand, deals only with the revenue needs of provinces.

The aim of this paper is to examine the efficiency and welfare effects of these equalisation schemes. This is achieved by developing a model with the same basic framework as the one constructed by Boadway and Flatters (1982). It is a model of a two-State federation where each State provides a local public good using a lump sum residence based tax. Citizens, the supply of labour to each State, are perfectly mobile domestically and make their location choice to satisfy an equal per capita utility condition. We argue that the location decisions made by labour are inefficient for two reasons. First, migrants generate a benefit for residents in the State receiving them, a lower tax price as the tax base expands, and a cost for residents in the State they leave, a higher tax price as the tax base contracts. Residents who migrate across States to equate their per capita utility may not take these 'fiscal externalities' into account. Second, if economic rents arising from the exploitation of natural resources are collected publicly and disbursed to citizens on the basis of residency, then labour may migrate to capture a share of these rents.

The next step is to show that there is an optimal inter-State transfer (the transfer could be mandated by a central authority) that establishes a globally efficient outcome. The transfer corrects for the externalities and location specific rents. It is the well-known transfer derived by Boadway and Flatters. Once the optimal transfer is implemented, the outcome is shown to be on the utility possibilities frontier defined between States. It must also be an outcome in which per capita utilities are equated across States (this is ensured by free migration). Thus, in this world one can have a globally optimal decentralised equilibrium that replicates the theoretically optimal centralised outcome.

To this point the analysis is standard. The innovation of the paper is to embed within the basic model a fiscal equalisation scheme. The scheme chosen is the Australian system of fiscal equalisation (implemented by the Commonwealth Grants Commission). This extension allows one to characterise an 'equalisation game' between States and examine the efficiency and welfare properties of a Nash equilibrium. The properties of this equilibrium can then be compared with the properties of the equilibrium from the basic federalism model with the efficient transfer implemented. The comparison leads to conclusions about the welfare properties of equalisation.

Specifically, it is shown that a Nash equilibrium of the equalisation game is inefficient because the inter-State transfer is not the one required on efficiency grounds and because provision of local public goods, and hence the tax decisions of

States, are inefficient. Both inefficiencies arise because the equalisation formulas tie States together in such a way that gives them an incentive to pursue self-interested and ultimately inefficient policies. Thus, while the outcome of the equalisation game is also on the 45 degrees line from the origin to the efficiency frontier defined between States, it is inside the frontier. Since the outcome from the basic federalism model with the efficient transfer implemented is also on the 45 degrees line, but on the efficiency frontier, this allows us to conclude that equalisation, at least as practiced in Australia, unambiguously lowers social welfare. It follows from this that any improvements to the current system of transfers that make it operate more like an efficiency-based system, that is, one that corrects for externalities, will improve social welfare.

The paper outline is as follows. Section 2 develops the basic model of a federal economy with sub-national regions (States), examines its efficiency properties and derives the efficiency enhancing inter-State transfer. Section 3 extends the basic model to incorporate the Australian equalisation model and characterises the equalisation game. The efficiency of the equilibrium to this game is examined here. Section 4 examines the welfare properties of the equilibrium with equalisation and compares these properties with those that emerge from the basic federalism model with the efficient transfer implemented. Conclusions are presented in Section 5.

2. The Basic Federalism Game

The federalism model developed is one of a country in which there are many sub-national regions, called States. Each State has a population of citizens who are identical (in terms of income and preferences) and supply the labour (along with a capital input) to produce a numeraire private good. This numeraire is consumed directly by citizens of the State or used to produce a single local public good, which is in turn consumed. The Government of each State chooses the amount of public good to be supplied and consumed, based on optimising behaviour.

At the outset it is useful to note that models of fiscal competition usually assume that there are two sub-national governments who are considered as players in a simultaneous move game with continuous pure strategies. Usually, each player has only one strategic variable, either a tax/subsidy or the level of expenditure on a public good, although models have emerged in which players have access to both strategic variables. The approach here adopts these features of the standard model[3].

2.1 Model Set Up

To focus attention on the essential problem, it is supposed that there are only two States in the model, denoted by i,j = 1,2. In State i there are n_i citizens who are identical in terms of income and preferences. The (fixed) national population is

[3] For example, see Boadway and Flatters (1982), Wildasin (1988), Wildasin (1991a), Wildasin (1991b), Wildasin and Wilson (1991), Myers (1990), Mansoorian and Myers (1993), Burbidge and Myers (1994), Wellisch and Wildasin (1996)). For an overview of this literature, see Wellisch (2000).

$$N = n_i + n_j . \qquad (1)$$

The production process in each State is extremely simple. Namely, each citizen supplies one unit of labour to the labour market of the State they reside in so that n_i is also a State's labour supply (N is the fixed national supply of labour). As explained below, labour is mobile between States so its supply can vary from the perspective of each State. There is also a vector of spatially immobile factors, denoted k_i, which might be thought of as public infrastructure, land or natural resources. The mobile and spatially immobile inputs are used in State i to produce a single output that serves as the numeraire for the State using the production function $f_i(n_i, k_i)$, assumed to be homogeneous of degree one. Combined with the assumption that factor markets are competitive, and that factors receive their marginal product, this implies that output is exactly exhausted by factor payments[4].

Since n_i is the only variable input the production technology of State i is

$$f_i(n_i) . \qquad (2)$$

where $f_i^{'}(n_i) > 0$, $f_i^{''}(n_i) < 0$. National output is simply the sum of State specific outputs, namely, $f = f_i(n_i) + f_j(n_j)$.

Because competitive factor markets are assumed, each citizen of State i receives a wage, w_i, that is equal to their marginal product. Since citizens of a State are identical, each receives the same wage, though wages may differ across States. It is also assumed that spatially immobile factors in State i are owned collectively by the residents of the State. Factor payments accruing to these inputs are collected publicly (by State i) and distributed as income to residents on the basis of residency. Each resident is assumed to receive an equal per capita share of these factor payments. Since the production function in State i is homogeneous of degree one, the implication is that the income of a citizen is simply the average product of the State[5]

$$\frac{f_i(n_i)}{n_i} . \qquad (3)$$

Each citizen of State i also has identical preferences defined over a single private good, denoted x_i, and a pure local public good, q_i. The utility function for a

---

[4] We abstract from international trade issues and suppose that the numeraire is traded on world markets at an exogenously given world price of one unit.

[5] This is a common feature built into models in the federalism literature, for example, see the papers listed in footnote 2. Though an abstraction, it is a simple device used to incorporate the idea that rents earned from the exploitation of location specific factors of production (eg. natural resources such as oil, gold etc) may be collected locally and used to benefit local residents. This in turn implies, as will be seen below, that migration decisions are influenced by these factor payments which can differ across States depending on the geographic distribution of spatially immobile factors.

representative resident, assumed to be quasi-concave, continuous and differentiable, is

$$u_i(x_i, q_i). \qquad (4)$$

The budget constraint of a representative citizen is

$$x_i + \frac{p_i q_i}{n_i} = \frac{f_i(n_i)}{n_i} \qquad (5)$$

where $p_i q_i / n_i$ is the equal per capita tax contribution that is used to fund the provision of the local public good. Rewriting (5), private good consumption for a representative citizen in State i is equal to their after tax (net) income (average product less their tax contribution),

$$x_i = \frac{f_i(n_i) - p_i q_i}{n_i}. \qquad (6)$$

As noted earlier, citizens are perfectly mobile between States, though not internationally[6]. They make their location choices to satisfy the equal utility condition, $u_i(x_i, q_i) = u_j(x_j, q_j)$. Using (6), and an equivalent expression for State j, the equal utility condition is

$$u\left(\frac{f_i(n_i) - p_i q_i}{n_i}, q_i\right) = u\left(\frac{f_j(n_j) - p_j q_j}{n_j}, q_j\right). \qquad (7)$$

Notice that we allow citizens to migrate in response to differences in after tax (net) income across States, but also to differences in the level of provision of the public good. Also, an allocation of labour across States that satisfies (7) will not in general equate wage rates or even per capita income.

2.2 Political Optimisation

It remains to specify how the Government of a State chooses its level of public good provision, and hence the tax payment to be made by citizens. There are various possibilities here. For example, one might suppose that political parties in each State choose public expenditure to maximise their chances of being elected. Alternatively, it is possible to suppose that State Governments pursue the interests of bureaucrats, the median voter or the numerical majority. Here we suppose that Governments are benevolent in the sense that they choose their public expenditures to maximise per capita utility of a representative resident within their jurisdiction.

[6] One could extend the analysis here to allow for international migration in response to State policies as in Shapiro and Petchey (2001) and Petchey and Shapiro (2001). This extension will lead to a more complex optimal transfer (see discussion below) but is unlikely to change the basic results.

Given that all citizens are identical in terms of preferences and incomes, this is equivalent to a median voter or majoritarian outcome.

In pursuing this political objective, State i must take into account its citizens' budget constraint and the possibility that labour will allocate itself across States to satisfy the equal utility condition. Therefore, the political problem of State i is to choose q_i to maximise

$$u_i\left(\frac{f_i(n_i)-p_i q_i}{n_i}, q_i\right) \qquad (8)$$

subject to the labour supply and migration constraints given by (1) and (7). The choice of q_i by State i affects the welfare of State j citizens because it influences, through the equal utility condition, the geographic distribution of labour, and hence the supply of labour and output (per capita income) in State j. Hence, States will act strategically and the problem can be characterised as a single period simultaneous move game with pure strategies. Nash conjectures are assumed, implying that each State chooses its expenditure policy to maximise its own citizen welfare, conditional on the policy chosen by its neighbour. It is supposed that in making their choices, States take account of the impact of their decisions on migration and the distribution of labour across States. An alternative is to assume that States are myopic and make their choices in ignorance of the migration responses to their decisions.

The best reply function for State i is found by differentiating (8) with respect to q_i (for given q_j) to obtain,

$$n_i mrs^i_{xq} + (w_i - x_i)\frac{\partial n_i}{\partial q_i} - p_i = 0 \qquad (9)$$

where $mrs^i_{xq} = u_{q_i} / u_{x_i}$ is the marginal rate of substitution between the public and private good in State i (the marginal benefit of an extra unit of the public good, in terms of consumption foregone). The migration response to a small increase in public good provision can be found by differentiating (1) and (7) totally with respect to q_i, for given q_j. In the matrix form $Ax = d$ this yields

$$\begin{pmatrix} \frac{u_{x_i}}{n_i}(w_i - x_i) & -\frac{u_{x_j}}{n_j}(w_j - x_j) \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{\partial n_i}{\partial q_i} \\ \frac{\partial n_j}{\partial q_i} \end{pmatrix} = \begin{pmatrix} \frac{u_{x_i}}{n_i} p_i - u_{q_i} \\ 0 \end{pmatrix}. \qquad (10)$$

The migration response is

$$\frac{\partial n_i}{\partial q_i} = \left(\frac{p_i}{n_i} u_{x_i} - u_{q_i} \right) / D \tag{11}$$

where D is the determinant of (10)[7]. Inserting the migration response expression into the best reply function and simplifying yields

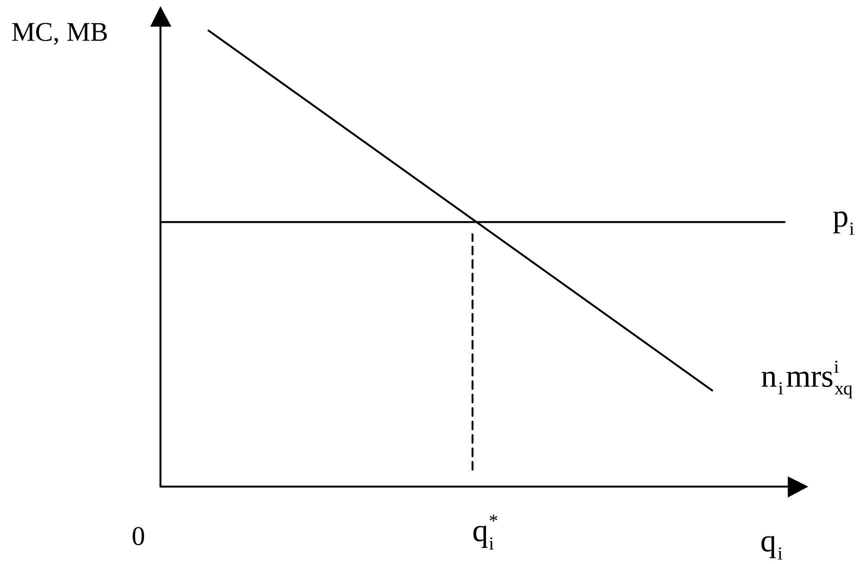
$$n_i mrs^i_{xq} = p_i . \tag{12}$$

Similarly, for State j, the best reply function is

$$n_j mrs^j_{xq} = p_j . \tag{13}$$

The Nash equilibrium to the game, assuming that one exists, is a policy pair, q_i^*, q_j^*, that solves (12) and (13) simultaneously[8]. The best reply functions are the well-known Samuelson rules for the efficient provision of pure public goods. They imply that a public good should be provided to the point where its total marginal benefit is equal to marginal cost. Since each State adopts this rule in the Nash equilibrium, provision of the public good is (locally) efficient. The equilibrium, from the perspective of State i, is characterised in Fig. 1 where the marginal rate of substitution (marginal benefit of the public good) and marginal cost are plotted.

Fig. 1: Equilibrium in State i



[7] Note that $D = \frac{u_{x_i}}{n_i}(w_i - x_i) + \frac{u_{x_j}}{n_j}(w_j - x_j)$.

[8] Boadway (1982) shows that (12) and (13) are the best reply functions even if States act myopically and ignore the migration responses to their decisions.

It should be noted that global efficiency requires that the Samuelson conditions be satisfied, and as shown below, that factors be allocated efficiently in a spatial sense. In the discussion of Section 2.3 it is shown that the latter condition is not satisfied in the Nash equilibrium. Therefore, the equilibrium is not globally efficient. However, it has been referred to above as locally efficient since the Samuelson conditions are satisfied. Thus, the equilibrium is inside the Utility Possibilities Frontier (UPF) defined between a representative individual from each State but on the 45 degrees line from the origin to the efficiency frontier (since free migration ensures that $u_i = u_j$ is satisfied in equilibrium).

2.3 The Optimal Inter-State Transfer

The efficiency consequences of the externalities noted above can be explored in more detail by observing that the equilibrium policy pair determines, from (7), the distribution of labour across States, and hence the supply of labour to each State. Thus, we can write $n_i(q_i, q_j)$. In this equilibrium, the net benefit to State i from accepting an additional migrant is $nb_i = w_i - x_i$. This is just the difference between what the migrant consumes and their wage or marginal product. For State j the net benefit is $nb_j = w_j - x_j$. Efficiency in the allocation of labour requires that the net benefits be equated in equilibrium, that is, $(w_i - x_i) = (w_j - x_j)$. It is well known (Boadway and Flatters (1982)) that the equal net benefit condition is not satisfied in a Nash equilibrium such as the one characterised above. This implies that the equilibrium distribution of labour across States is inefficient. There is an optimal transfer from State i to j, denoted as ϕ, that corrects for the inefficiency and establishes an efficient geographic distribution of labour. The optimal transfer is the value of ϕ that solves $(w_i - x_i - \phi) = (w_j - x_j + \phi)$. This is,

$$\phi^{opt} = \frac{1}{2}\Big((w_i - x_i) - (w_j - x_j)\Big). \tag{14}$$

The optimal transfer can also be expressed in terms of externalities, as in Boadway and Flatters (1982) and Petchey (1993, 1995), by substituting equation (6), and an equivalent expression for State j, into (14) and rearranging. This yields[9],

$$\phi^{opt} = \frac{1}{2}\left(\left(\frac{p_i q_i}{n_i} - \frac{p_j q_j}{n_j}\right) - \left(R_i - R_j\right)\right). \tag{15}$$

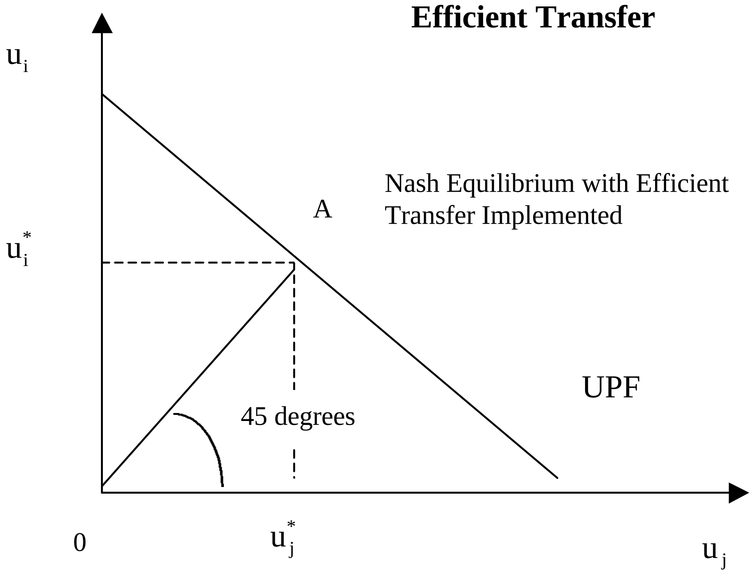
where $R_i = (f_i(n_i)/n_i) - w_i$ is the per capita share of a citizen of State i in that State's spatially immobile factor income (we can think of this as a citizen's share of the economic rent in State i) and $R_j = (f_j(n_j)/n_j) - w_j$ is the per capita share of a

[9] Boadway and Flatters (1982) argue that the optimal transfer should be mandated by a central authority. Myers (1990) shows that States will voluntarily choose the optimal transfer making the Nash equilibrium globally efficient. Shapiro and Petchey (2000) generalise this idea using a 'coincidence theorem'. The result is explained in Attachment A.

citizen of State j in that State's immobile factor income (a citizen's share of State j rent)[10].

If a central authority, for example the Commonwealth Grants Commission in Australia, implemented an inter-State transfer consistent with ϕ^{opt}, then on the assumption that States optimise taking the efficient transfer as given, the outcome of the game described above will be globally efficient and on the UPF defined between a representative individual from each State at a point such as A (Fig. 2)[11]. Notice that the outcome must also be on the 45 degrees line from the origin to the efficiency frontier since $u_i = u_j$ must be satisfied in equilibrium.

Fig. 2: Outcome of the Game with the Efficient Transfer



To implement the efficient inter-State transfer, the central authority would need to estimate fiscal externalities and externalities generated by the consumption of location specific economic rents by migrating labour. It is also worth noting that there may be other types of externality that distort the free migration equilibrium, for example, location specific amenities of the type modeled in Petchey and Shapiro (2001). Moreover, one could allow the public good characterised here to be crowded simply by introducing a crowding parameter. At one extreme value of the parameter the public good would be pure (the case characterised here) and at the other extreme the good would be purely private. In the latter case, the per capita tax terms disappear from the optimal inter-State transfer condition and the efficient

[10] In Petchey and Shapiro (2001), and Shapiro and Petchey (2001), citizens do not capture a share of State specific spatially immobile factor income. Hence, in the models in those papers, the efficient transfer is the one that ensures equal per capita tax contributions across States. That is, the transfer corrects only for the fiscal externalities.

[11] The possibility that States will take into account the effects of their policy decisions on the optimal transfer (ie. 'game' against the efficient transfer formula) is not one that is considered in the federalism literature. If States did act in this way, it is possible that the equilibrium with the optimal transfer in place would be inefficient, as States would no longer follow the Samuelson rule (though the allocation of labour across States would be efficient).

transfer need only correct for location specific economic rents captured by migrating labour (or location specific amenities if they were included). All these possibilities could be included, but whatever mix of externalities one models, the end result that there is an efficient transfer that correct for the externalities, is unchanged.

The exception of course is the one where there are no externalities that distort migration decisions. In this case, the equilibrium to the game is globally efficient since labour is always allocated efficiently across States. Moreover, any inter-State transfer in this case will create efficiency costs, since the optimal transfer is zero. This seems to be the case modeled in some computable general equilibrium applications to fiscal federalism, for example, the one undertaken in Groenewold, Hagger and Madden (2001a).

3. An Equalisation Game

The basic federalism model developed above is now extended to incorporate a fiscal equalisation scheme. This allows one to examine a richer ‘equalisation game’ in which States choose their policies while being linked together through an equalisation procedure. By deriving the conditions that must hold in a Nash equilibrium of this game, and comparing these conditions with those that must hold in an efficient outcome, we are able to draw conclusions about the efficiency of public good provision and the inter-State allocation of labour in a federation where States are linked through a centrally mandated equalisation scheme. Conclusions about the welfare effects of equalisation also follow.

It should be noted before proceeding that if we were interested only in the efficiency of the inter-State allocation of labour under equalisation, one would not need to characterise such a game and derive the Nash conditions. Rather, all that is required is that we examine the transfer under equalisation and compare it with the optimal transfer derived previously. Since we can surmise in advance that the two will differ (since the optimal transfer is based on efficiency and the equalisation transfer is based on equity considerations), it is unlikely that the equalisation transfer will be efficient regardless of what optimising process States use to choose their policies. However, since we are interested primarily in the impact of equalisation on the provision of public goods, it is necessary to characterise the full equalisation game, find the Nash conditions and see whether there is anything in these conditions that ensures that the Samuelson conditions are met. In this way, the efficiency of public good provision and the allocation of labour are determined simultaneously: one issue cannot be separated from the other.

In this regard, suppose that there is a national government that levies an identical lump sum tax, denoted s, on each person in the federation. The total pool of revenue created, G, must, therefore, be equal to sN. For simplicity, we assume that s is given[12]. Since N is fixed, this implies that the pool of revenue, G = sN, is

[12] The determination of s could be made endogenous by explicitly modelling national government optimising behaviour. If States take account of the impact of their policy decisions on s then additional distortions, not directly related to equalisation, would be introduced to the model. We abstract from these considerations by supposing that States treat s as given.

also fixed. Suppose also that the resident of State i obtains a per capita amount (a grant) from this pool, denoted as g_i. The per capita budget constraint is now

$$x_i + s + \frac{p_i q_i}{n_i} - g_i = \frac{f_i(n_i)}{n_i}. \qquad (16)$$

3.1 Australia's Fiscal Equalisation Scheme

The way in which g_i is determined must now be modeled in detail. The option chosen is the Australian fiscal equalisation model (developed and maintained by the Commonwealth Grants Commission) mainly because it is comprehensive, rigorous and well documented. The aim of this scheme, in common with other equalisation schemes around the world, is to achieve an equity goal, specifically, to allow each State in the Australian federation to provide some standard level of public services (eg. health, education, law and order and public transport) while imposing a standard tax burden on their citizens. It does this by compensating States for so-called cost and revenue 'disabilities' that they may face when providing public services and using these disabilities to calculate expenditure and revenue 'needs'.

A complete specification of the model is presented in Attachment B. From there it can be seen that g_i is given by

$$g_i = \frac{G}{N} + \frac{E}{N}(\gamma_i - 1) + \frac{T}{N}(1 - \rho_i), \qquad (17)$$

where $E = p_1 q_1 + p_2 q_2$ (total public expenditure by the States), $T = E - G$ is the revenue raised by the States to finance public expenditure, γ_i is an 'expenditure disability' and ρ_i is a 'revenue disability'. If the disabilities equal one, State i's per capita grant is equal to its per capita share of the pool, G/N. If either one, or both, of the disabilities deviates from one, the State's per capita grant also deviates from its equal per capita share. The direction of the deviation depends on the direction in which the disabilities deviate from one. For example, State i may have expenditure and revenue disabilities ($\gamma > 1, \rho < 1$), an expenditure disability and revenue advantage ($\gamma > 1, \rho > 1$), an expenditure advantage and revenue disability ($\gamma < 1, \rho < 1$) or both revenue and expenditure advantages ($\gamma < 1, \rho > 1$). The magnitude and sign of the disabilities determine whether the per capita grant is greater than or less than the State's equal per capita share of the pool.

Note also that, as can be seen in Attachment 2, the cost disability is a parameter that does not vary with changes in public expenditure and regional population. On the other hand, the revenue disability does vary with changes in public expenditure and regional populations. Of course, if we allowed for something other than constant costs associated with the public good, then the cost disability would be a function of the level of public expenditure and regional populations. This is a possibility that we do not explore in this paper.

Suppose also, consistent with observed practice, that the pool is fully exhausted by grants to the States, that is,

$$n_i g_i + n_j g_j = G . \qquad (18)$$

3.2 The Inter-State Transfer

What is the inter-State transfer that results from the application of such a grant procedure? For given values of $s, p_i, p_j, \rho_i, \gamma_i$ and N, the per capita grant to State i is a function of combined State policy choices,

$$g_i(q_i, q_j) . \qquad (19)$$

Moreover, States will generally not receive a grant that is equal to their contribution through the national tax. The implication is that equalisation redistributes income between States. Denote the per capita transfer to (from) State i as $\phi_i = g_i - s$. The inter-State transfer is endogenous since it is a function of the per capita grant, which is in turn a function of collective policies, and s, the national government tax used to create the pool.

If the per capita grant received by State i from the equalisation scheme, g_i, is exactly equal to what it contributes to the pool, s, then there is no inter-State transfer and $\phi_i = 0$. If $g_i > s$ then $\phi_i > 0$ and State i receives more from the pool than it contributes (a transfer in its favour). When $g_i < s$ we know that $\phi_i < 0$ and State i contributes more to the pool than it receives. Similarly, denote any per capita transfer from (to) State j as $\phi_j = g_j - s$. We also know that $\phi_i = -\phi_j$ or $\phi_j = -\phi_i$. Hence, a transfer to (from) State i is $\phi_i = -(g_j - s)$.

Recall from the above discussion that the State specific grant is a function of State policies. If we consider the equal utility migration condition, (7), one can also see that the distribution of labour across States is also a function of State policies, $n_i(q_i, q_j)$. Therefore, the inter-State transfer from i to j is also a function of combined policies,

$$\phi_i(q_i, q_j) . \qquad (20)$$

3.3 Nash Conditions

As in the game of Section 2, suppose that States choose public expenditures to maximise the per capita utility of a representative resident within their jurisdiction. In pursuing this, States take into account their citizens' budget constraints, the equalisation formulas and the migration condition.

Once again, State i's choice of public good provision affects State j's welfare through the equal utility condition. But there is an additional source of interdependence through the equalisation formulas. State i's choice of public good provision also affects its grant, and hence the grant of the neighbouring State (and hence welfare in that State). Supposing Nash conjectures, State i makes its policy choice conditional on the choice of State j. However, it is assumed that the State takes into account the migration response to its choices, and the grant response, both for itself and for its neighbour. These assumptions imply that States do not take their transfer as given when making their expenditure and tax choices.

As in Section 2, States can be thought of as playing a simultaneous-move game in pure strategies, but this is now thought of as an 'equalisation game' in which the problem of State i is to choose q_i (given q_j) to maximise

$$u_i(x_i, q_i)$$

subject to the budget, equalisation and equal utility constraints,

$$\begin{aligned}
&\text{(i)}\ \ x_i + s + \frac{p_i q_i}{n_i} - g_i = \frac{f_i(n_i)}{n_i} \\
&\text{(ii)}\ \ g_i = \frac{G}{N} + \frac{E}{N}(\gamma_i - 1) + \frac{T}{N}(1 - \rho_i) \\
&\text{(iii)}\ \ n_i g_i + n_j g_j = G \\
&\text{(iv)}\ \ u_i(x_i, q_i) = u_j(x_j, q_j) \\
&\text{(v)}\ \ n_i + n_j = N
\end{aligned} \tag{21}$$

Before proceeding to characterise optimal policy choices, it should be noted that the way the problem is set up, with governments facing an equal utility migration condition, the transfers that arise from equalisation cannot alter relative utilities across States. In other words, if equalisation has the objective of raising utility in one State relative to another, this objective cannot be achieved because free migration always ensures that utilities are equal between States. The only way that equalisation could be successful in its objective is if there is some 'attachment to home' on the part of citizens which allows a wedge to persist in comparative inter-State utilities (labour is no longer perfectly mobile). This possibility is not explored here.

Substituting constraint (i) into the utility function and differentiating with respect to q_i (for given q_j) yields the best reply function for State i,

$$n_i mrs^i_{xq} + \frac{\partial n_i}{\partial q_i}\left(w_i - x_i - s + g_i\right) + n_i \frac{\partial g_i}{\partial q_i} - p_i = 0. \tag{22}$$

This is analogous to (12), the best reply function in the game without equalisation. But now there is an extra term that captures the change in the (per capita) grant in State i that results from a small increase in provision of the public good.

The expression for the migration response term can be found by writing the budget constraint for each State in terms of x_i and x_j, substituting into constraint (iv), and, along with the total labour supply condition, differentiating totally with respect to q_i (given q_j). This yields

$$\begin{pmatrix} \frac{u_{x_i}}{n_i}(w_i - x_i - s + g_i) & -\frac{u_{x_j}}{n_j}(w_j - x_j - s + g_j) \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{\partial n_i}{\partial q_i} \\ \frac{\partial n_j}{\partial q_i} \end{pmatrix} = \begin{pmatrix} \frac{u_{x_i}}{n_i} p_i - u_{q_i} - \theta_i \\ 0 \end{pmatrix} \quad (23)$$

and

$$\frac{\partial n_i}{\partial q_i} = \left(u_{x_i} \frac{p_i}{n_i} - u_{q_i} - \theta_i \right) / D \quad (24)$$

where D is the determinant in (23) and $\theta_i = u_{x_i} \frac{\partial g_i}{\partial q_i} - u_{x_j} \frac{\partial g_j}{\partial q_i}$. Comparing this with the migration response function without equalisation or inter-State transfers, (11), the difference is that (24) includes θ_i in the numerator. This captures the impact of equalisation on the migration response to changes in public good provision in State i.

The grant response term, $\partial g_i / \partial q_i$, is found by differentiating constraint (iii) in (21) yielding

$$\frac{\partial g_i}{\partial q_i} = -\frac{n_j}{n_i} \frac{\partial g_j}{\partial q_i} - \frac{\partial n_i}{\partial q_i} \left(\frac{g_i + g_j}{n_i} \right). \quad (25)$$

Writing (17) in terms of State j, and differentiating with respect to q_i (given q_j) yields

$$\frac{\partial g_j}{\partial q_i} = \frac{p_i}{N}(\gamma_j - \rho_j) - \frac{\partial \rho_j}{\partial q_i} \frac{T}{N}. \quad (26)$$

From (A5) in Attachment B[13],

$$\frac{\partial \rho_j}{\partial q_i} = -\frac{\partial n_i}{\partial q_i} \left(\frac{N w_j + \rho_j((w_i - w_j)n_j - f)}{f \cdot n_j} \right). \quad (27)$$

A system of Nash conditions analogous to (22), and (24) to (27), applies for the other player, State j. Together, the Nash conditions of the two States comprise a system of simultaneous equations in the unknowns[14]

[13] Recall the definition $f = f_i(n_i) + f_j(n_j)$ where f is total national output (income).

$$q_i, \frac{\partial g_i}{\partial q_i}, \frac{\partial g_j}{\partial q_i}, \frac{\partial \rho_j}{\partial q_i}, \frac{\partial n_i}{\partial q_i}, q_j, \frac{\partial g_j}{\partial q_j}, \frac{\partial g_i}{\partial q_j}, \frac{\partial \rho_i}{\partial q_j}, \frac{\partial n_j}{\partial q_j}. \qquad (28)$$

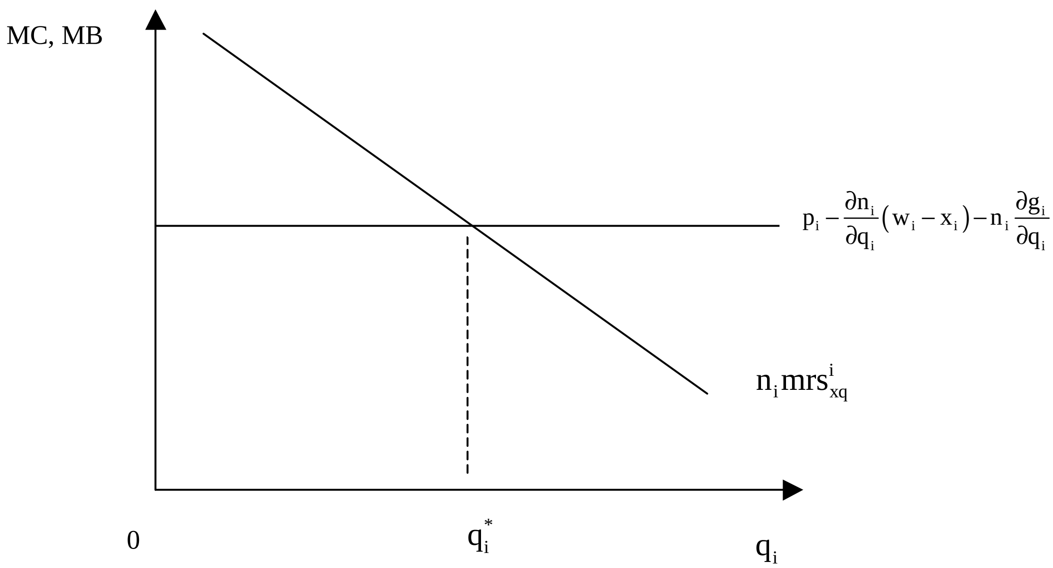
3.4 Efficiency

A Nash equilibrium of the equalisation game, assuming one exists[15], is a policy pair q_i^*, q_j^*, which solves the system of Nash conditions. Given the complexity of the system, it is not possible to solve analytically for the equilibrium policy choices[16]. Nevertheless, one can draw some conclusions about the efficiency of the equilibrium without explicit solution, either analytic or numerical.

The first efficiency result relates to public good provision in each State. In this regard, expressions (12) and (13) are the conditions that must be satisfied for locally efficient provision of the public goods. In a Nash equilibrium of the revenue sharing/equalisation game, (22) and (24) to (27), together with equivalent conditions for State j, are satisfied (simultaneously). Clearly, the policy pair that solves this system of equations is, in general, not the policy pair that solves the Samuelson conditions. Therefore, the equilibrium policy choices from the revenue sharing/equalisation game yield inefficient supplies of the local public goods.

The equilibrium level of provision of the public good in the equalisation game, in terms of marginal cost and marginal benefit, is characterised in Fig. 3.

Fig. 3: Equilibrium (for State i) in the Equalisation Game



---

[14] The parameters are k_i, k_j, k, N, p_i, p_j and s.

[15] Work is progressing on an existence proof. The possibility of multiple equilibria must be considered.

[16] Numerical solution is possible and requires use of the definitional equations for $g_1, g_2, x_1, x_2, \gamma_1, \gamma_2, \rho_1, \rho_2$, E and T.

Notice that State i equates marginal benefit with the underlying marginal cost of provision, p_i (as in the game with no equalisation), but that now the underlying marginal cost is adjusted by the migration and grant responses to changes in public good provision. Depending on the sign of these responses, the 'adjusted' marginal cost will be above or below the underlying marginal cost. If it is below, then State i's provision of the public good will tend to be pushed up by the presence of mobility and equalisation, while if it is above, State i's provision of the public good will tend to be pushed down by mobility and equalisation.

The second efficiency relates to the inter-State allocation of labour. In this regard, the equilibrium (per capita) transfer from State i to j in the equalisation game is $\phi_i^* = -(g_j^* - s)$. Using (17) this becomes

$$\phi_i^* = s - \frac{1}{N}\left(G\rho_j + E^*(\gamma_j - \rho_j)\right) \qquad (29)$$

where E^* is the total State expenditure on public goods associated with the equilibrium levels of provision. Since it takes no account of fiscal externalities or location specific economics rents, the factors that are important from an efficiency perspective, this expression will, in general, result in an inter-State transfer that differs from ϕ_i^{opt}, the transfer required for efficiency. Therefore,

$$\phi_i^* \neq \phi^{opt}, \qquad (30)$$

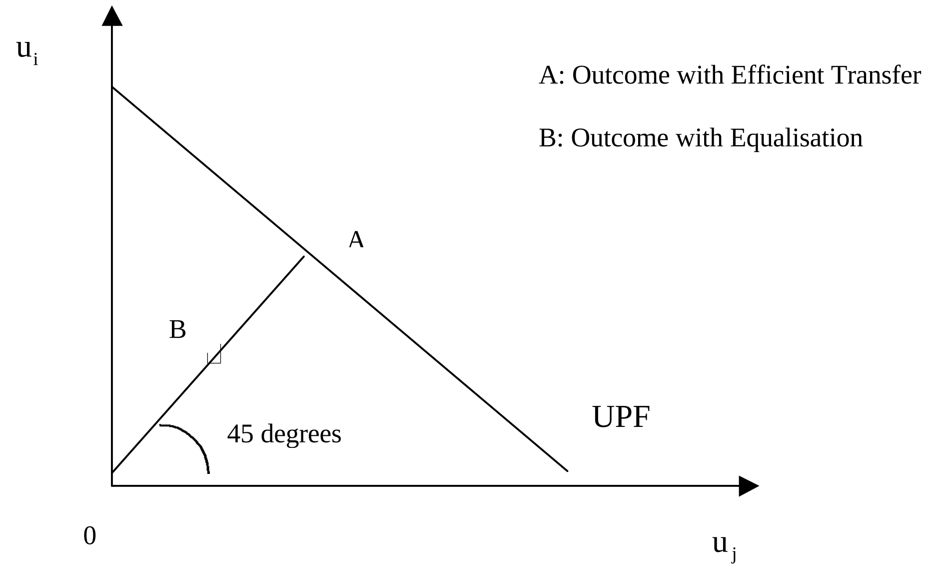
and the Nash equilibrium of the game yields an inefficient spatial labour distribution. When States choose the inter-State transfer indirectly, via their choice of per capita grant within an equalisation scheme, the transfer chosen is not the one required to establish an efficient allocation of labour. Note that while we have demonstrated the inefficiency of the transfer in a Nash equilibrium (the policy choices that appear in (29) are Nash policies), the conclusion about the inefficiency of the transfer holds regardless of the optimising process States follow to choose their policies. In other words, it is likely to be the case that the transfer would be inefficient if one characterised an equilibrium based on different behavioural assumptions.

Thus, the equilibrium of the equalisation game is inefficient for two reasons: the allocation of labour across States is inefficient and States do not adopt the Samuelson rule for the provision of public goods.

4. Welfare

In Fig. 4, the outcome from the game of Section 2, with the efficient transfer implemented, is characterised as point A. Notice that the outcome from this game must be on the 45 degree line (from the origin to point A) where the free migration condition, $u_i = u_j$, holds. It is also on the efficiency frontier because it is globally optimal. The outcome from the revenue sharing/equalisation game is at a point such as B that must also lie on the 45 degrees line. However, from the discussion above we know that it must lie inside the efficiency frontier because of the distortions associated with an inefficient inter-State transfer and levels of provision of the public good that do not satisfy the Samuelson condition.

Fig. 4: Welfare Effects of Equalisation



It is clear that equalisation makes both States unambiguously worse off relative to an outcome in which the efficient inter-State transfer is implemented. Equal per capita utility with equalisation (point B) is always lower than equal per capita utility with the efficient transfer implemented (point A)[17]. Note also that equalisation does not achieve its stated equity effects because even the recipient State (either i or j in our two State model) is worse off (as well as the contributing State). In other words, equalisation cannot succeed in raising utility in one State relative to another because States are tied together through mobility (we must always have $u_i = u_j$ satisfied). In addition to this, equalisation creates inefficiency in both the distribution of mobile factors and the provision of public goods.

Note that one can consider $u_i = u_j$ to be the national social welfare function, W. Hence, define

$$W = u_i = u_j . \tag{31}$$

If the current equalisation scheme were to be modified and developed into a scheme that is based on an efficient transfer, there would is a clear gain in social welfare (W must rise as will the welfare of residents in both States). Indeed, the outcome is one that is consistent with a social optimum with a utilitarian social

[17] The magnitude of the distance between A and B is an empirical issue. Work is also progressing on constructing a version of the equalisation game that adopts specific functional forms and allows numerical solution to find equilibrium values for the endogenous variables. This will allow simulations to be undertaken to gain some idea of the likely significance of any welfare losses associated with equalisation.

welfare function. Thus, the outcome with the efficient transfer is not only globally efficient; it is also equitable. On the other hand, the current system is inefficient, and cannot achieve its Stated equity goals because of the presence of free migration that makes it impossible to change relative utilities across States. All that the current system does is achieve a lower level of equal per capita welfare[18].

5. Conclusions

The early Sections of the paper developed a model of a two-State federation with labour (citizen) mobility. States engaged in a game in which they provided local public goods by applying a residence based tax on labour. It was shown that States would provide public goods in a way that is locally efficient but that the distribution of citizens across States would be inefficient. The inter-State transfer needed to establish an efficient distribution of labour was then derived. It was shown the outcome of the game is globally efficient if States choose their levels of public good provision (and taxes) with the efficient transfer being implemented by a central authority. Such an outcome is also equitable in the sense that per capita utilities are equated across States.

An equalisation game was then characterised in which States choose their levels of provision of a public good (and hence own source taxes) while taking account of migration responses to their choices as well as changes in their equalisation grant. It was shown that an equilibrium to this game, while again yielding equal per capita utilities, is inefficient for two reasons: the Samuelson condition for the efficient provision of public goods is violated and the inter-State transfer is inefficient. Equalisation was also shown to lead to an unambiguously lower level of social welfare without achieving its stated equity objectives.

The analysis leads to the conclusion that if the Australian equalisation model was reformed to reflect more of the factors that one should be concerned about on efficiency grounds (eg. fiscal externalities, location specific rents, general externalities and congestion costs), then the reforms would unambiguously improve social welfare and be equitable. To refocus the transfer system in this way, one would need to measure fiscal externalities, location specific economic rents, and any other externalities considered appropriate, in order to construct ϕ^{opt}. The empirical feasibility of this is a matter for further investigation.

---

[18] Interestingly, Petchey (2001) shows that national political parties engaged in a vote maximising game of political competition would choose an efficient transfers regime in equilibrium, that is, one that secures an outcome such as A in Fig. 4.

Attachment A: The Coincidence Result

As noted in the main text, Myers (1990) argues that if States are able to make direct lump sum transfers, they will choose the optimal transfer, as well as provide public goods efficiently. The Nash equilibrium is globally efficient and there is no need for a central authority. Shapiro and Petchey (2000) show that the Myers' result is an example of a more general 'coincidence theorem'. To explain the idea in the context of the current discussion, one can depict the problem of State i in Section 2 of the text in a slightly different way. Specifically, the set of instruments available to State i is expanded to include a lump sum inter-State transfer. With the transfer included, the problem of State i is to choose q_i and ϕ to maximise

$$u_i\left(\frac{f_i(n_i)-p_iq_i-\phi}{n_i},q_i\right). \tag{A1}$$

subject to the constraints (1) and (7) in the main text (with the transfer incorporated into (7)). The best reply condition for the public good is once again, expression (12) in the main text[19]. To find the necessary condition for the transfer, differentiate (A1) with respect to ϕ and obtain

$$(w_i - x_i)\frac{\partial n_i}{\partial \phi} - 1 = 0\,. \tag{A2}$$

Differentiating (1) and (7) in the main text totally with respect to ϕ (given q_i) yields

$$\begin{pmatrix} \frac{u_{x_i}}{n_i}(w_i - x_i) & -\frac{u_{x_j}}{n_j}(w_j - x_j) \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \frac{\partial n_i}{\partial \phi} \\ \frac{\partial n_j}{\partial \phi} \end{pmatrix} = \begin{pmatrix} \frac{u_{x_i}}{n_i} - \frac{u_{x_j}}{n_j} \\ 0 \end{pmatrix}$$

and

$$\frac{\partial n_i}{\partial \phi} = \left(\frac{u_{x_i}}{n_i} - \frac{u_{x_j}}{n_j}\right)/D\,, \tag{A3}$$

where D is the determinant (as in footnote 6). Inserting (A3) into (A2) yields

$$(w_i - x_i) = (w_j - x_j)\,. \tag{A4}$$

---

[19] Unlike the case with no voluntary State transfers, where provision was locally efficient, here provision of the public good is globally efficient.

Attachment B: Equalisation Model

The Australian equalisation model, developed by the Commonwealth Grants Commission (CGC)[20], obtains an expression for g_i by first examining the expenditure and revenue of the states using so-called disabilities, and then constructing a formula for the division of a given pool G based on the expenditure and revenue needs of the States. The following discussion presents a detailed analysis of each step taken and is, therefore, intended as background discussion to the main text which shows how the expression for g_i is derived. More generally, it would be possible to use other national revenue sharing models, instead of the Australian model, for the determination of g_i.

I. Expenditure

In examining expenditure, the CGC first defines the total expenditure of all States as (inclusive of any grant received)

$$E = p_i q_i + p_j q_j , \tag{A1}$$

and per capita expenditure (known as standard expenditure) as E/N. The Commission also estimates 'cost disabilities' that measure the extent to which the marginal cost of providing services in one State deviates from the average marginal cost due to factors such as population dispersion, economies of scale, ethnic background of the population and age/sex composition. The cost disability for State i is

$$\gamma_i = \frac{2p_i}{p_i + p_j} . \tag{A2}$$

The cost disability, and State specific population, are applied to E/N to derive

$$n_i \frac{E}{N} \gamma_i . \tag{A3}$$

This expression (known as standardised expenditure) yields the amount that State i would need to spend if it is to provide the level of service associated with the standard (average) achieved by all States. It is above standard expenditure if the State has relatively high costs of provision ($\gamma_i > 1$). Alternatively, if $\gamma_i < 1$, State i can spend less than the average in order to obtain the average level of service.

II. Revenue

With regard to revenue, the CGC calculates the total tax revenue raised directly by all States, exclusive of the equalisation grant received. This is known as

[20] The analysis in this Attachment is based on the model presented in Commonwealth Grants Commission (1999).

‘total own source revenue’, denoted from now on as T. It is equal to the total expenditure of all states, E, less the grant pool, G (since all of the grant pool is distributed, as is shown below),

$$T = E - G. \qquad \text{(A4)}$$

Next, the CGC estimates T / N, the (per capita) own-source revenue of the States and calls this standard revenue. A ‘revenue disability’ for State i is then estimated which captures the extent to which per capita income in State i deviates from average per capita income for all States. The revenue disability is[21],

$$\rho_i = \frac{Nf_i(n_i)}{fn_i}. \qquad \text{(A5)}$$

The last step is to calculate total standardised revenue,

$$n_i \frac{T}{N} \rho_i. \qquad \text{(A6)}$$

This is an estimate of the revenue that State i will raise if it makes the average tax effort. If the State has relatively low per capita income ($\rho_i < 1$) then standardised revenue is less than standard revenue. Therefore, if it makes the average tax effort, the State will raise less than standard revenue. Alternatively, if State i has average per capita income ($\rho_i = 1$) then standardised and standard revenue are identical. Finally, if the State has above average per capita income ($\rho_i > 1$) then standardised revenue is greater than standard revenue. In this case, if it makes the average tax effort, it raises more than standard revenue.

III. The Main Formulas

Having developed the building blocks, it is now possible to construct the main formulas that determine the relativity, ψ_i. The process starts with the Total Financing Requirement of State i, denoted $TFAR_i$. This is equal to total standardised expenditure less total standardised revenue,

$$TFAR_i = n_i \frac{E}{N} \cdot \gamma_1 - n_i \frac{T}{N} \cdot \rho_1 \quad . \qquad \text{(A7)}$$

The $TFAR_i$ can also be written as

[21] In practice, the CGC estimates a separate disability for each State specific tax base. We use (A5), which is based on State output, as a proxy for these base -specific disabilities for the sake of simplicity without any loss of meaning for the purposes of our exercise. Recall also the definition $f = f_i(n_i) + f_j(n_j)$.

$$\mathrm{TFAR}_i = n_i \left(\frac{E-R}{N} + \frac{E}{N} \cdot (\gamma_i - 1) + \frac{T}{N} \cdot (1-\rho_i) \right). \qquad \text{(A8)}$$

From (A4), it is known that $G = E - T$ so that (A8) can be expressed as

$$\mathrm{TFAR}_i = n_i \left(\frac{G}{N} + \frac{E}{N} \cdot (\gamma_i - 1) + \frac{T}{N} \cdot (1-\rho_i) \right) \qquad \text{(A9)}$$

The last piece of the jigsaw is to estimate the relativity of State i. The Commission defines state i's per capita relativity, ψ_i, as

$$\psi_i = \frac{\mathrm{TFAR}_i}{n_i} / \frac{G}{N}. \qquad \text{(A10)}$$

Substituting (A9) into (A10) and simplifying, the per capita relativity becomes

$$\psi_i = 1 + \frac{E}{G}(\gamma_i - 1) + \frac{T}{G}(1-\rho_i). \qquad \text{(A11)}$$

The State specific grant is

$$g_i = \psi_i \frac{G}{N}. \qquad \text{(A12)}$$

Substituting (A11) into (A12) and solving yields

$$g_i = \frac{G}{N} + \frac{E}{N}(\gamma_i - 1) + \frac{T}{N}(1-\rho_i). \qquad \text{(A13)}$$

This is the equation for the per capita grant used in the main text.

References

Boadway, R.W., and F. Flatters (1982). Efficiency and Equalization Payments in a Federal System of Government: A Synthesis and Extension of Recent Results. *Canadian Journal of Economics.* 15: 613-33.

Boadway (1982). On the Method of Taxation and the Provision of Local Public Goods: Comment. *American Economic Review*. 72. 846-51.

Burbidge, J. B., and G.M. Myers (1994). Redistribution Within and Across the Regions of a Federation. *Canadian Journal of Economics.* 27:620-36.

Commonwealth Grants Commission (1999). Report on General Revenue Grant Relativities, Vol. II: Methods, Assessments and Analysis. Commonwealth of Australia. Canberra.

Dixon, P.B., Madden, J.R and Peter, M.W. (1993). The Effects of Reallocating General Revenue Assistance Among the Australian States. *The Economic Record.* 69:207, 367-81.

Groenewold, N., Hagger, A.J. and J.R. Madden (2001a). The Effects of Inter-Regional Transfers With Optimising Regional Governments. *Paper Presented at the Fourth Annual Conference on Global Economic Analysis.* June 27-29.

Groenewold, N., Hagger, A.J. and J.R. Madden (2001b). Competitive Federalism: A Political-Economy General Equilibrium Approach. *Paper Presented to the 24th Annual Conference of the Australia and New Zealand Regional Science Association International,* Hobart, 3-6 December.

Mansoorian, A., and G.M Myers (1993). Attachment to home and efficient purchases of population in a fiscal externality economy. *Journal of Public Economics.* 52:117-32.

Myers, G.M (1990). Optimality, Free Mobility and the Regional Authority in a Federation. *Journal of Public Economics.* 43:107-21.

Petchey, J.D (1993). Equalisation in a Federal Economy With Inter-State Migration. *Australian Economic Papers.* December. 336-54.

Petchey, J.D (1995). Resource Rents, Cost Differences and Fiscal Equalisation. *The Economic Record.* 71:215. December. 343-53.

Petchey, J.D (2001). Political Competition and Fiscal Equalisation. *Australian Journal of Political Science.* 36:2, 309-23.

Petchey, J.D and P. Shapiro (2001). State Tax and Policy Competition for Mobile Capital. *The Economic Record (Forthcoming).*

Shapiro, P., and J.D Petchey (2000). The Coincidence of Collective and Individual Interests. *Mimeograph.*

Shapiro, P., and J.D Petchey (2001). Internationally Mobile Factors and Economic Policy in an Integrated Regional Union of States. *Mimeograph.*

Wellisch, D., and D.E. Wildasin (1996). Decentralized income redistribution and Immigration. *European Economic Review*. 40: 187-217.

Wellisch, D., (2000). *Theory of Public Finance in a Federal State*. Cambridge University Press.

Wildasin, D.E. (1988). Nash Equilibrium in Models of Fiscal Competition. *Journal of Public Economic.* 35: 229-40.

Wildasin, D.E. (1991a). Income Redistribution in a Common Labor Market. *American Economic Review*. 81: 757-74

Wildasin, D.E. (1991b). Some Rudimentary 'Duopolity' Theory. *Regional Science and Urban Economics.* 21:393-421.

Wildasin, D.E., and J.D. Wilson (1991). Theoretical Issues in Local Public Economics. *Regional Science and Urban Economics.* 21: 317-31.